Math 121 Homework Assignment #1

Spring 2007

Due Friday April 13

From your textbook:

Page 8 # 7,8,11

Page 12-13 #7

Also:

1. Show that the completion of $\mathbb{Q}$ in the sense of problem 7 (on page 12-13) is $\mathbb{R}$.

2. Show that the irrational numbers in $\mathbb{R}$ are not a countable union of closed sets.

Deduce that the rational numbers are not a countable intersection of open sets.

3. A condensation point of a set S in a metric space X is (by definition) a point p such that, for each $\varepsilon>0$, $S \cap B(p, \varepsilon)$ is uncountable (uncountably infinite).

Prove: If S is an uncountable set in $\mathbb{R}$ (or more generally in $\mathbb{R}^{\mathbf{n}}$) then there exists a condensation point for S.

> *Suggestion*: $\mathbb{R}=\bigcup_{-\infty}^{\infty}[n, n+1]$, so there is some interval[n,n+1] such that $[n,n+1]\cap S$ is uncountable. Hence, either $[n,n+\frac{1}{2}]\cap S$ or $[n+\frac{1}{2},n+1]\cap S$ is uncountable. Continue to find sequence of "nested" intervals of length $\frac{1}{2^k}$, k=1,2,3… each of which has an uncountable intersection with S.